THERMAL STRESSES DUE TO INTERNAL FRICTION OF THE TUBE OF A PNEUMATIC TIRE CLUTCH

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A method is examined of determining the design thermal stresses due to friction of the material during operation of a pneumatic tire clutch. An example is given of the calculation of the thermal stresses in the tube of a PM 300 × 100 clutch.

The thermophysical properties of the rubber-cord material have an appreciable influence on the thermal stresses in the tube of a tire clutch. For rubber-cord material the thermal conductivity is 300 times less, and the thermal expansion is 20 times greater than for steel. The thermal conductivity of the tube material depends on the filler, carbon black, and may vary over a range of five orders of magnitude.

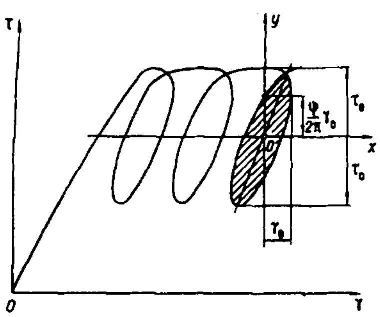

Fig. 1. Shape of elliptical hysteresis loop.

Formulation and solution of the heat conduction equation. The following assumptions are made:

1) The tube is cylindrical in shape.

2) There is no heat flux in the direction of the tube axis, and the tube gives off heat to the environment from its inner and outer faces at a temperature constant with time (Θ_1 and Θ_2).

3) The tube material is homogeneous and isotropic.

4) The thermal conductivity of the rubber-cord material is constant and independent of temperature.

Under these assumptions, the equation of steady heat conduction of a uniform cylinder with an internal heat source will have the form

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + q\frac{1}{c\gamma} = 0. \qquad (1)$$

All the parameters appearing in (1), except q, are known. We shall deal in more detail with determination of q.

The section most stressed in a thermal sense is the top cylindrical layer of the cover, where the displacements are greatest, while the thermal conduction in comparison with the cord is least. This is confirmed by numerous tests conducted in the LVVMIU (Leningrad Higher Naval Engineering College) Strength and Friction Laboratory [1]. However, in practice breakdown of the tube occurs in the lower cylindrical layer, where the stresses are greatest, and stress concentration occurs at the places of transition from the round to the cylindrical part of the tube. Moreover, the lower friction layer is subjected to additional heating from external friction when the clutch and its loads are engaged and disengaged.

The magnitude of the energy dissipation in internal friction of the tube material arising from cyclic loads on the clutch may be evaluated from the hysteresis loop. It should be noted, however, that there are as yet no reliable theoretical or experimental data to describe the shape of the hysteresis loop, i.e., the law of variation of the forces of internal inelastic resistance in the process of cyclic deformation. We shall therefore assume an elliptical hysteresis loop (Fig. 1), which is confirmed by the investigations [1]. According to [2], the relation between the shearing stress $\tilde{\tau}$ and the shear angle γ resulting from the action of internal friction in the tube is given by the expression

$$\tau = G\left[\gamma + \frac{\psi}{2\pi}\gamma_0\sqrt{1-\left(\frac{\gamma}{\gamma_0}\right)^2}\right]. \qquad (2)$$

Omitting simple transformations, we may write

$$\Delta w = \frac{1}{2}\psi G\beta^2 r^2. \qquad (3)$$

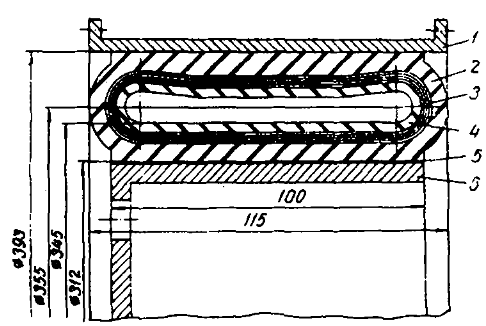

Fig. 2. The PM 300 × 100 pneumatic tire clutch: 1) outer metal cylinder; 2) rubber cover; 3) rubber-cord body; 4) rubber chamber; 5) friction layer; 6) inner metal drum.

Setting the energy dissipated per cycle equal to the amount of heat generated in unit volume of the tube material per unit time, with sufficient accuracy for practical purposes, we may write

$$q = \frac{1}{2}\psi G\beta^2 r^2 n. \tag{4}$$

Introducing the notation

$$K = \frac{1}{2}\psi G\beta^2 n, \tag{5}$$

we obtain

$$q = Kr^2, \tag{6}$$

and then the heat conduction equation (1) is rewritten as

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + K\frac{r^2}{c\gamma} = 0. \tag{7}$$

Relative to the problem being examined

$$\frac{dt}{dr} = \alpha_1 (t - \Theta_1) \quad \text{when} \quad r = r_1,$$
$$\frac{dt}{dr} = -\alpha_2 (t - \Theta_2) \quad \text{when} \quad r = r_2; \tag{8}$$

$$t = \Theta_2 + \frac{Kr_2^4}{16\lambda}(1 - \mu^4) - \left(K\frac{r_2^4 - r_1^4}{16\lambda} + \Delta\Theta\right)\frac{\ln\mu}{\ln\mu_0}. \tag{9}$$

Here the values of Θ_2 and Θ_1 are defined as

$$\Theta_1 = \frac{1}{\xi}\Big\{\alpha_2 r_2 \Theta_2' + \alpha_1 r_1 (1 - \alpha_2 r_2 \ln\mu_0)\Theta_1' + \frac{K}{16\lambda}[4\alpha_2 r_2 r_1^4 \ln\mu_0 + (4 + \alpha_2 r_2)(r_2^4 - r_1^4)]\Big\},$$

$$\Theta_2 = \frac{1}{\xi}\Big\{\alpha_1 r_1 \Theta_1' + \alpha_2 r_2 (1 - \alpha_1 r_1 \ln\mu_0)\Theta_2' - \frac{K}{16\lambda}[4\alpha_1 r_1 r_2^4 \ln\mu_0 - (4 - \alpha_1 r_1)(r_2^4 - r_1^4)]\Big\}, \tag{10}$$

where

$$\xi = \alpha_1 r_1 + \alpha_2 r_2 - \alpha_1\alpha_2 r_1 r_2 \ln\mu_0,$$

$$\Delta\Theta = -\frac{1}{\xi}\Big\{r_1 r_2 \ln\mu_0\left[\alpha_1\alpha_2\Delta\Theta' + \frac{K}{4\lambda}(\alpha_1 r_2^3 + \alpha_2 r_1^3)\right] + \frac{K}{16\lambda}(\alpha_1 r_1 + \alpha_2 r_2)(r_2^4 - r_1^4)\Big\}.$$

$$\Delta\Theta' = \Theta_2' - \Theta_1'.$$

The thermal stress equation and its solution. To determine the thermal stresses due to internal friction in the clutch tube, we make use of the equations of [3]:

$$\sigma_1 = \frac{\alpha_3 E}{1 - \nu}\Big\{-\frac{Kr_2^4}{96\lambda}\left[1 - (1 + \mu_0^2)(1 + \mu^2)\frac{\mu_0^2}{\mu^2} - 5\mu^4\right] +$$

$$+ \frac{1}{2}\left[\Delta\Theta\frac{Kr_2^4}{16\lambda}(1 - \mu_0^4)\right]\left[\left(\frac{1 + \ln\mu}{\ln\mu_0} + \frac{1 + \mu^2\mu_0^2}{1 - \mu^2\mu_0^2}\right)\right]\Big\}, \tag{11}$$

$$\sigma_2 = \frac{\alpha_3 E}{1 - \nu}\Big\{-\frac{Kr_2^4}{48\lambda}(1 + \mu_0^2 + \mu_0^4 - 3\mu^4) + \frac{1}{2}\Big[\Delta\Theta + \frac{Kr_2^4}{16\lambda}(1 - \mu_0^4)\Big]\left[\left(\frac{1 + 2\ln\mu}{\ln\mu_0} + \frac{2\mu_0^2}{1 - \mu_0^2}\right)\right]\Big\}. \tag{12}$$

A test of operation and a number of laboratory studies show that rupture of the clutch tubes occurs where the temperatures in the body of the tube attain their greatest values, i.e., on the outer and inner faces of the tube. Taking into account that, in addition, the mechanical stresses from the cyclic loads reach their greatest values on the inner and outer faces of the tube, it becomes clear that the extreme values of both stresses should be expected also on the above-mentioned surfaces of the clutch tube.

The simultaneous action of these stresses will predetermine the beginning of fatigue rupture on the outer and inner faces of the clutch tube.

We shall examine the thermal stresses during cyclic deformation of the tube as a function of the temperature drop $\Delta\Theta$ and the dimensionless parameters $\mu = 1$ and $\mu_0 = r_1/r_2$.

From (11) and (12) we have

$$\sigma_1 = \sigma_2 = \frac{\alpha_3 E}{1 - \nu}\Big\{\frac{Kr_2^4}{24\lambda}\left(1 + \mu_0^2 + \mu_0^4 + \frac{3}{4}\frac{1 - \mu_0^4}{\ln\mu_0}\right) + \Delta\Theta\left(\frac{\mu_0^2}{1 - \mu_0^2} + \frac{1}{2\ln\mu_0}\right)\Big\}; \tag{13}$$

$$\sigma_1 = \sigma_2 = \frac{\alpha_3 E}{1 - \nu}\Big\{\frac{Kr_2^4}{24\lambda}\left(1 + \mu_0^2 + \mu_0^4 + \frac{3}{4}\frac{1 - \mu_0^4}{\ln\mu_0}\right) + \Delta\Theta\left(\frac{1}{1 - \mu_0^2} + \frac{1}{2\ln\mu_0}\right)\Big\}. \tag{14}$$

Example. We give below the results of calculations of the thermal stresses in the tube of a PM 300 × × 100 pneumatic tire clutch (Fig. 2) with the following data: $\alpha_3 = 6.7 \cdot 10^{-4}$; $E = 2.94$ MN/m^2; $\nu = 0.5$; $\psi =$ = 0.2 and 0.3; $G = 0.981$ MN/m^2; $\beta = 0.084$ rad; $\omega =$ = 16.6 sec^{-1}; $\tau' = 4$ hr; $n = 24 \cdot 10^4$; $r_2 = 196$ mm; $\lambda = 0.162$ W/m · degree; $\mu \approx 0.8$; $\Delta\Theta = \Theta_2 - \Theta_1 = 60°$.

According to (13)

when $\psi = 0.2$ $\sigma_1 = \sigma_2 = -0.56$ MN/m^2,
when $\psi = 0.3$ $\sigma_1 = \sigma_2 = -0.84$ MN/m^2.

According to (14)

when $\psi = 0.2$ $\sigma_1 = \sigma_2 = 1.08$ MN/m^2,
when $\psi = 0.3$ $\sigma_1 = \sigma_2 = 1.58$ MN/m^2.

NOTATION

θ_1, θ_2) temperatures on inner and outer faces of the tube; r) variable radius; c) specific heat; γ') specific weight of tube material; q) internal source of heat generated by internal friction; τ) shear stress; γ) shear angle; G) shear modulus of material; ψ) coefficient of energy dissipation per cycle; γ_0) amplitude of shear angle; $\Delta\omega$) element of work of energy dissipation per cycle; β) relative angle of twist; n) number of loading cycles; $\mu = r/r_2$, $\mu_0 = r_1/r_2$) dimensionless parameters; r_1, r_2) inner and outer radii of tube; α_1, α_2) relative heat transfer coefficients; σ_1) normal tangential stress; σ_2) normal axial stress; α_3) coefficient of thermal expansion; ν) Poisson's ratio; E) Young's modulus; ω) angular frequency; τ') testing time; λ) thermal conductivity.

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